Jacard distance between two sets defined as

$$d_J(A,B) = 1 - \frac{|A \cap B|}{|A \cup B|}$$

Note:1 $0 \leq d_J \leq 1$.

Note :2

If $|A \cap B| = 0$, then $d_J(A,B) = 1$

Note 3: If $|A \cap B| = |A \cup B|$, then $d_J(A,B) = 0$.

Problem:

Find the Jacard Distance between the two sets $A = \{0, 1, 2, 5, 6\}$ $B = \{0, 2, 3, 5, 7, 9\}$

solution:

$$d_J(A,B) = 1 - \frac{|A \cap B|}{|A \cup B|}$$

$A \cap B = \{0, 2, 5\} \Rightarrow |A \cap B| = 3$

$A \cup B = \{0, 1, 2, 3, 5, 6, 7, 9\} \Rightarrow |A \cup B| = 8$

$\therefore d_J(A,B) = 1 - \frac{3}{8} = 0.625$

(ii) What will happen when adding 7 to A

Solution:

$A' = \{0, 1, 2, 5, 6, 7\}$

$B = \{0, 2, 3, 5, 7, 9\}$

Now, $d_J(A', B) = 1 - \frac{|A' \cap B|}{|A' \cup B|}$.

$A' \cap B = \{0, 2, 5, 7\} = 4$.

$A' \cup B = \{0, 1, 2, 3, 5, 6, 7, 9\} = 8$

Now, $d_J(A', B) = 1 - \frac{4}{8} = 1 - \frac{1}{2} = 0.5$

Inference:

$d_J(A, B) = 0.625$

$d_J(A', B) = 0.5$

$\Rightarrow d_J(A', B) < d_J(A, B)$

$\Rightarrow$ The distance between the sets A and B are closer to each other.

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(iii) What will happen when adding 4 to A.

$A'' = \{0, 1, 2, 4, 5, 6\}$, $B = \{0, 2, 3, 5, 7, 9\}$

Now, $A'' \cap B = \{0, 2, 5\}$ $|A'' \cap B| = 3$.

$A'' \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 9\} = 8$

$$d_J(A'', B) = 1 - \frac{|A'' \cap B|}{|A'' \cup B|}$$

$$= 1 - \frac{3}{9}$$

$$= 1 - 0.33$$

$$= 0.67.$$

Now,

$d_J(A, B) = 0.625$ and

$d_J(A'', B) = 0.67$

$\Rightarrow$ $d_J(A, B) < d_J(A'', B)$

$\Rightarrow$ Distance between the sets A and B are farther from each other.